A ROBUST IMAGE SHARPNESS METRIC BASED ON KURTOSIS MEASUREMENT OF WAVELET COEFFICIENTS

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ABSTRACT

In this paper, a robust sharpness metric for noisy images is presented. The proposed method, relying on the Lipschitz regularity properties separating the signal singularities from the noise singularities, applies the dyadic wavelet transform and then measures the kurtosis of the two-dimensional discrete Fourier transform of the wavelet coefficients across large scales. In contrast to other sharpness metrics, the proposed technique will perform well under low to moderate SNR since the noise will be reduced across wavelet scales. Simulation results reveal that the proposed image sharpness metric is less sensitive to noise as compared to existing sharpness metrics.

1. INTRODUCTION

Recently, research focusing on non-reference objective quality metrics gained considerable attention [1, 2, 3]. For example, a noise metric can be used to estimate the quantization error caused by compression without accessing either the original pictures or the bitstream, while a sharpness metric can be used as a control parameter for sharpness enhancement algorithms applied to digital imagery. Sharpness metrics can also be combined with other metrics to compute overall quality. In this paper, we propose a sharpness metric that can be used for noisy as well as clean images. The metric could be used in digital video as well as in fully automated systems such as focusing scanning electron microscopes (SEM). Note that SEM images are noisy by nature due to the statistical nature of electron collision and emission as well as the final detection system [4].

Many sharpness metrics for clean images were proposed. Some rely on spatial techniques such as gradients, cellular logic, spectrum analysis, histogram thresholding, image correlation, and image variance. A review of sharpness metrics applied to optical microscopy images is presented in [5]; these methods can be easily extended to natural images. Batten [6] investigated the performance of these metrics under noisy SEM images and concluded that gradient-based methods seem to be the most susceptible to noise, while the spectral methods and variance both seem to be quite robust to noise when dealing with SEM images. An advantage of the variance measure over the spectral methods is that it requires less computations.

A statistical measure based on the bivariate kurtosis was proposed in Zhang et al. [1]. The Discrete Fourier Transform (DFT) is computed first for the whole image, and the resultant matrix is treated as a two-dimensional probability density function that is used to compute the kurtosis. Caviedes and Oberti [1] computed sharpness using the average 2D kurtosis of the $8 \times 8$ Discrete Cosine Transform (DCT) blocks of the edge regions.

However, all the above mentioned metrics are sensitive to noise and will fail under. For example, the variance metric will work on noisy SEM images since they all have approximately the same amount of noise; nevertheless, the variance metric fails if the noise between the compared images is varied. Measuring the sharpness of images having different amount of noise is not a trivial task.

This paper extends Zhang’s method [3] to the wavelet domain. The Lipschitz property, proved by Mallat [7], indicates that noise is reduced as the scale is increase in the wavelet transform domain. The noise will be taken to be as an additive Gaussian noise throughout all conducted simulations.

The proposed method is presented in Section 3. Simulation results and experiments performed are presented in Section 4. Future directions is given in Section 5.

2. KURTOSIS

In this section, the univariate as well as the multivariate definition of kurtosis is reviewed. Application of the kurtosis as an image sharpness metric is also explained.

2.1. Definition of the kurtosis

Given a univariate random variable $X$ with mean $\mu$, and finite moments, the kurtosis is defined as a normalized 4th moment as follows:

$$k = \frac{E[(X - \mu)^4]}{[E[(X - \mu)^2]]^2}$$  \hspace{1cm} (1)

The kurtosis of a normal distribution is equal to +3 while that of a uniform one is equal to 1.8. If the distribution is concentrated entirely at its mean, the kurtosis approaches infinity. On the other hand, the kurtosis minimum is one and is achieved for a symmetrical two point distribution. Mardia [8] extended the kurtosis definition to multivariate random variables. Let $W$ be a $p$-dimensional random vector with finite moments up to the fourth, $\mu$ the mean vector and $\Gamma$ the covariance matrix, the kurtosis of $W$ is given by:

$$k_p = E[(W - \mu)^T \Gamma^{-1} (W - \mu)]^2$$  \hspace{1cm} (2)

When $p = 2$, the kurtosis can be evaluated as in [8]:

$$k_2 = \frac{\gamma_{4.0} + \gamma_{0.4} + 2\gamma_{2.2} + 4\rho_{1.2}(\rho_{2.2} - \gamma_{1.3} - \gamma_{3.1})}{(1 - \rho_{2.2}^2)^2}$$  \hspace{1cm} (3)
where $\rho_{12}$ is the correlation between the two random variables $W_1$ and $W_2$ and $\gamma_{kl}$, for $k, l = 0, 1, 2, 3, 4$, is calculated as:

$$\gamma_{kl} = \frac{E[(W_1 - \mu_1)^{2}(W_2 - \mu_2)^{2}]}{\sigma_1^2\sigma_2^2}$$

(4)

Note that $\mu_1$, $\mu_2$, $\sigma_1$, and $\sigma_2$ are the marginal means and marginal standard deviations of $W_1$ and $W_2$, respectively. Because of the averaging nature of moments, interpreting the kurtosis is not straightforward; attempts were made by statisticians to come up with a unified comprehensible explanation. Balanda al. [9] presented a review of the concept of kurtosis and concluded that the kurtosis is best defined as a measure of both peakedness near the center and tail weight of the distribution.

2.2. Kurtosis as an image sharpness metric

Zhang et al. [3] showed that a two-dimensional spectral density function can be treated as a probability density function. The two-dimensional spectral density of the image $f(x, y)$ of size $N \times N$ is estimated using the periodogram as:

$$|F(u, v)|^2 = \frac{1}{(2\pi N)^2} \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} f(x, y)e^{-j\frac{2\pi km}{N}} e^{-j\frac{2\pi ln}{N}}$$

(5)

where $F(u, v)$ denotes the discrete fourier transform. Using (3) the kurtosis is calculated after normalizing the result obtained from (5). The kurtosis, which in a sense, measures the width of the shoulder of the distribution, can be used as a sharpness metric. Sharp images tend to have higher frequency content due to the presence of edges and thus flatter distributions; thus, a smaller kurtosis indicates a sharper image. The kurtosis-based sharpness metrics were successfully used in [3, 1]. The metric was shown to be consistent with subjective quality and, thus, can be used as a control parameter in a feedback loop to adjust image sharpness. The problem occurs when noise is present in images. The kurtosis is applied on the vertical and horizontal detail subbands. The discrete dyadic wavelet transform (DDWT) is used due to its stability, completeness and ease of implementation by filter banks. Fig. 2 shows the flowchart of the proposed method.

3. PROPOSED METHOD

The proposed method is based on measuring the kurtosis on noisy images in the wavelet domain. The kurtosis is applied on the vertical and horizontal detail subbands. The discrete dyadic wavelet transform (DDWT) is used due to its stability, completeness and ease of implementation by filter banks. Fig. 2 shows the flowchart of the proposed method.

3.1. Discrete dyadic wavelet transform (DDWT)

The wavelet model is not required to have a continuous scale; rather, to allow numerical implementation, the scale will be restricted to vary along the dyadic sequence $2^j$. By applying the DDWT to the noisy image $g(u; v)$, two detailed-subband images $W^1_s(u; v; s)$ and $W^2_s(u; v; s)$ and one low-frequency subband image $W^0_s(u; v; s)$ are obtained at each scale $s = 2^j$. The DDWT can be implemented using a filter bank structure. Fig. 2 shows the one-level DDWT of a 2-D image signal $g(u; v)$. Only $W^2_s(u; v; s)$ will be used for decomposition at the next scale. A 3-level DDWT decomposition is performed in the proposed method. In our implementation, the used wavelet filter coefficients are the same ones used in [10].

3.2. Noise analysis using DDWT

The image information is separated from noise by trying to discriminate the edge singularities from the noise singularities. The Lipschitz exponents of the image singularities are measured from the evolution of the wavelet transform maxima across scales. When irregular textures are not present in the image, the majority of singularities are discontinuities. All singularities have positive Lipschitz exponents; so, the wavelet transform modulus maxima will not increase when the scale decreases. On the other hand, a white noise distribution is almost singular everywhere giving negative

Fig. 1. Image sharpness measurement using the kurtosis [3]. (a) Original image, kurtosis $= 4.48$; (b) Original image with additive gaussian noise with PSNR $= 8.22$ dB and kurtosis $= 5.54$; (c) Blurred noisy original image using a $7 \times 7$ averaging filter with PSNR $= 10.23$ dB and kurtosis $= 5.33$. 
Lipschitz exponents. Mallat et al. [7] proved that the average density function of the modulus $M$ of a two-dimensional real, wide-sense stationary, white noise random signal $n(x, y)$ having a variance $\sigma^2$ is:

$$E(|M_n(2^j, x, y)|^2) = \sigma^2 (\|\psi^1(x, y)\|^2 + \|\psi^2(x, y)\|^2) 2^j$$  \hspace{1cm} (6)

where $\psi^1(x, y)$ and $\psi^2(x, y)$ are two wavelets defined as the partial derivative of a smoothing function $\Theta_s(x, y)$ along $x$ and $y$, respectively. The two components of the wavelet transform are proportional to the coordinates of the gradient vector of $f(x, y)$ smoothed by $\Theta_s(x, y)$.

Knowing that the modulus maximum at a scale $2^j$ measures the derivative of the signal smoothed at $2^j$, one should be able to correlate that with the Lipschitz regularity of the signal. The evolution across scales of the wavelet transform depends on local Lipschitz regularity of the signal. A function $f(x)$ is uniformly Lipschitz $0 < \alpha < 1$ over an interval $[a, b]$ if and only if there exists a constant $K > 0$ such that for all $x \in [a, b]$, the wavelet transform satisfies [11]:

$$|W_{2^j} f(x)| \leq K(2^j)^\alpha$$  \hspace{1cm} (7)

$$\log_2 |W_{2^j} f(x)| \leq \log_2 (K) + \alpha j$$  \hspace{1cm} (8)

From (8), we can deduce that, if the Lipschitz regularity is positive, the amplitude of the wavelet transform modulus maxima should decrease when the scale decreases. In contrast, the wavelet transform maxima of a singularity increases with decreasing scale resulting in a negative Lipschitz exponent and indicating a singularity rather than a discontinuity. However, this evolution property of the amplitude of wavelets’ coefficients is not only valid for the maximum modulus points. Xu et al. [12] proposed a method to keep the significant wavelet coefficients that are due to edges and eliminate the noisy ones by interscale coefficients correlation analysis.

### 4. SIMULATION RESULTS

A set of experiments were performed to show the performance of the proposed wavelet-based kurtosis metric. To test the proposed method, we introduced different blur amounts by filtering the Elane image using a $7 \times 7$ Gaussian filter with a standard deviation equals to 0.8, 1.2, 1.6, 2.0 and 2.4, respectively. Noise is added to the blurred images using an additive Gaussian noise of zero mean and 0.02 variance. The added noise is quite noticeable on the image but, at the same time, it is not hindering the image characteristics. Fig. 4 shows a sample of the images under tests, while Fig. 5 shows the performance of the kurtosis metric as well as the proposed one when applied to these noisy images. Analyzing Fig. 5a, we can observe that the kurtosis-based metric is not able to distinguish the source of the high frequency components on the tail of the distribution whether it is due to edges or noise. Thus, for blurred image having a variance of 0.8 seems, for the metric, sharper than the one having a variance of 0.4. Note that the measured value from the kurtosis is inversely to obtain the metric. On the other hand, in Fig. 5b, the proposed wavelet-based metric is performing as expected; the metric is decreasing as the blurriness in the image is increasing.

### 5. FUTURE DIRECTIONS

Simulations results were also performed on different images having different amounts of blur. The objective was to try to investigate the performance of available sharpness metrics on different set of images having different characteristics. Fig. 5 shows the images used for testing. The images were chosen having different characteristics. For example, the 'Man' image is rich in textures while the 'Peppers' image contains large flat areas. The proposed method as well as the described ones are failing to predict the correct amount of blur when comparing for different images.

Preliminary subjective results were performed. The testing
Fig. 4. Images used in testing set, (a) Eliane image with blur variance = 0.4 and noise variance = 0.02, (b) Eliane image with blur variance = 2.4 and noise variance = 0.02.

Fig. 5. Performance of the kurtosis and proposed metrics in the presence of noise (a) Kurtosis, (b) Proposed method.
Fig. 6. Images used in subjective test, (a) Fishingboat, (b) Man, (c) Peppers, (d) Houses.

was done using SUN machines for their high quality displays. Each time, the subject was exposed to a combination of two images out of the four available in Fig. 5. The total number of combinations is equal to 6. Five non-professional subjects were picked to give their opinion; the subjects had to compare the two images in each combination and state which one is more blurred. Results reveal that the majority of the subjects were able to differentiate the level of blurriness between different images while none of the mentioned metrics, even the proposed one, performed well. A sharpness metric that is capable of predicting and comparing the sharpness of different images will be investigated in the future.

6. REFERENCES


